

Graph Abstraction

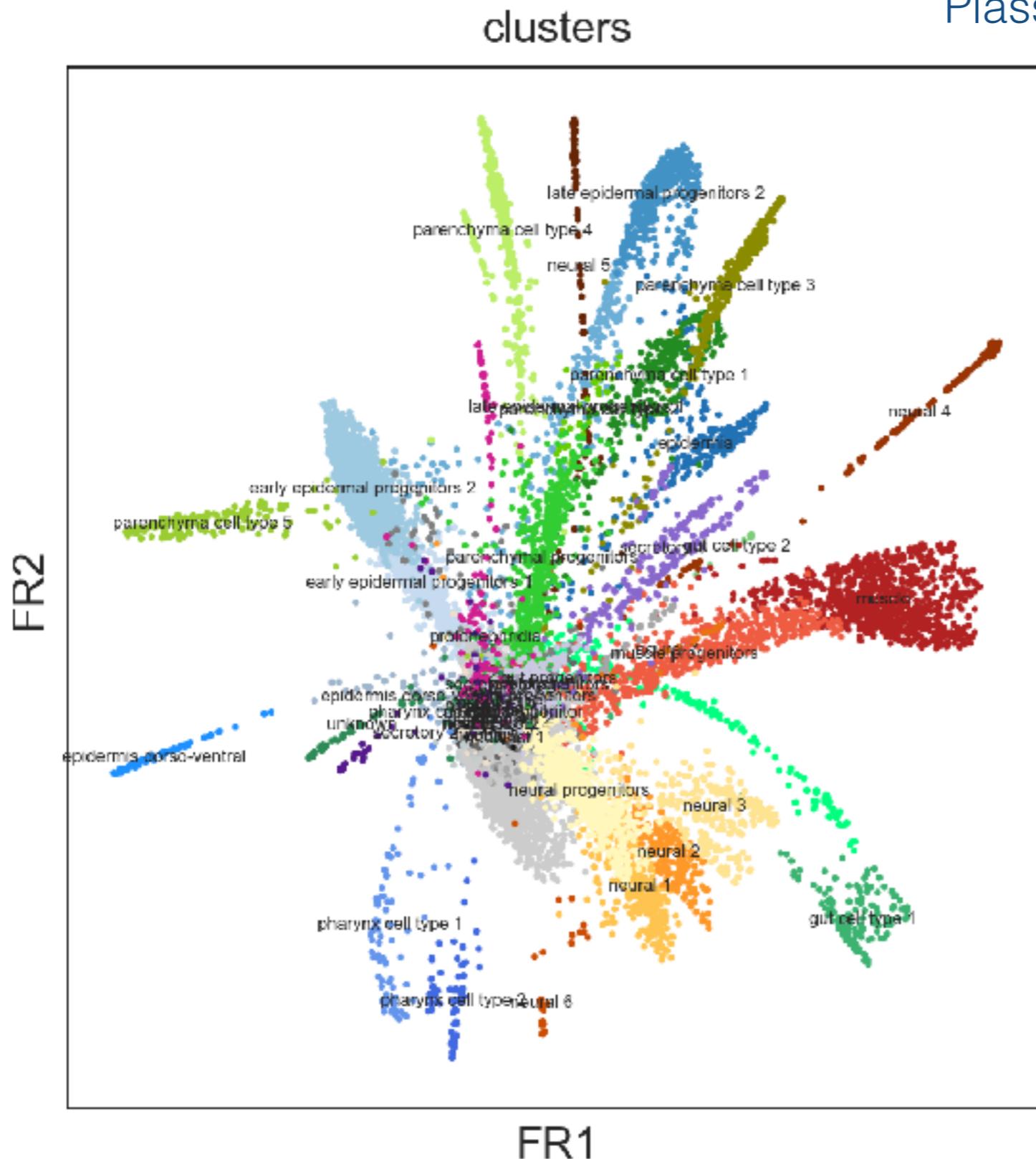
Reconciling clustering with trajectory inference through a topologically consistent map of single cells

F. Alexander Wolf, Institute of Computational Biology, Helmholtz Munich
October 17, 2017 - Single Cell Genomics - Weizmann Institute of Sciences

46 cell types of planaria



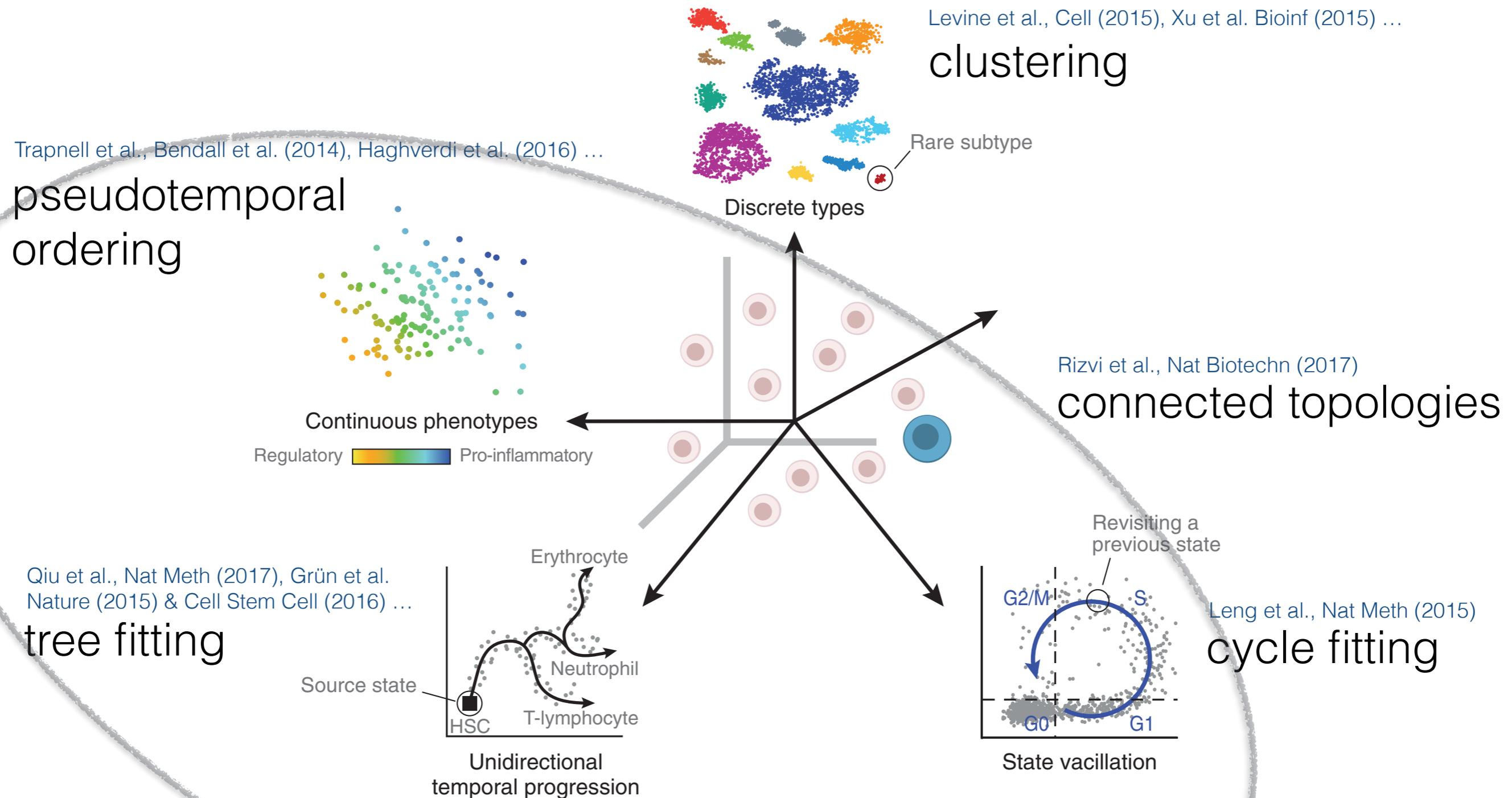
Plass, Solana, ..., Rajewski, unpublished (2017)



- Which “cell types”/ clusters are connected?
- Which paths do cells take, where do branchings occur?
- Trace gene “dynamics”/ changes along paths?

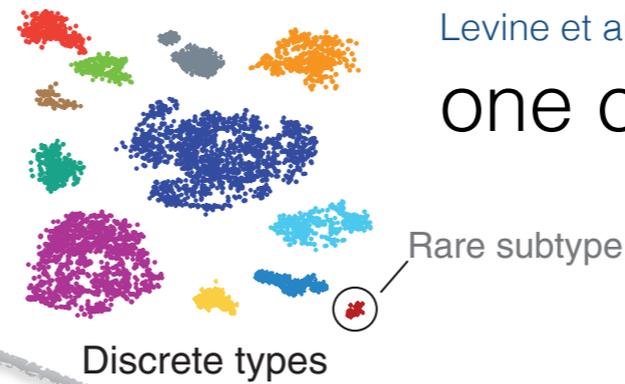
Weinreb et al., bioRxiv (2017)

Cell-to-cell variation: overview



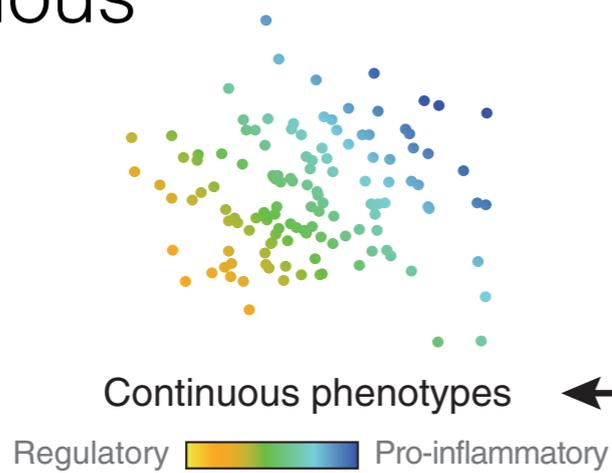
Cell-to-cell variation: coordinates

Levine et al., Cell (2015), Xu et al. Bioinf (2015) ...
one categorical coordinate

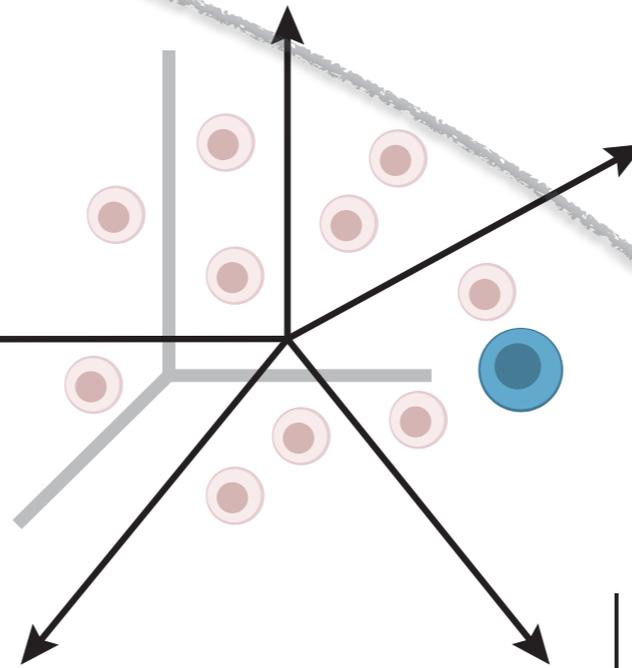


Trapnell et al., Bendall et al. (2014), Haghverdi et al. (2016) ...

one continuous coordinate

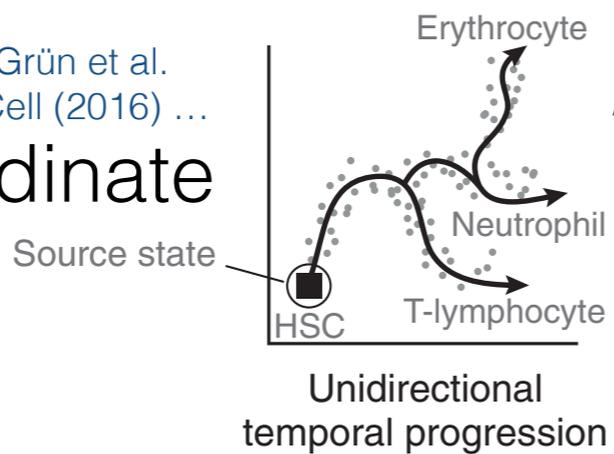


Rizvi et al., Nat Biotechn (2017)
connected-graph like coordinate system

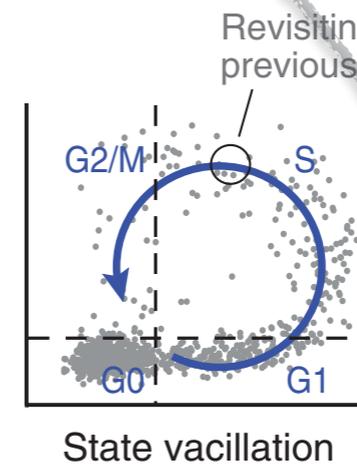


Qiu et al., Nat Meth (2017), Grün et al. Nature (2015) & Cell Stem Cell (2016) ...

tree-like coordinate system

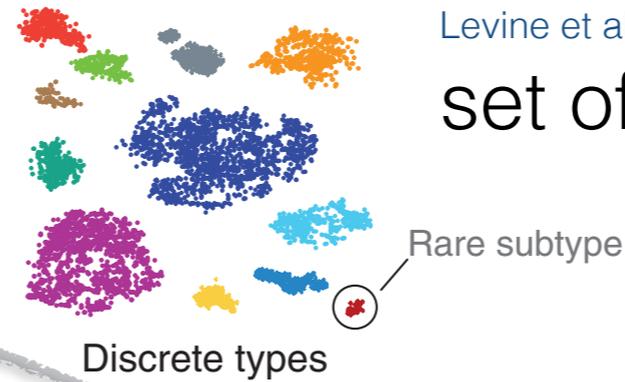


Leng et al., Nat Meth (2015)
continuous coordinate



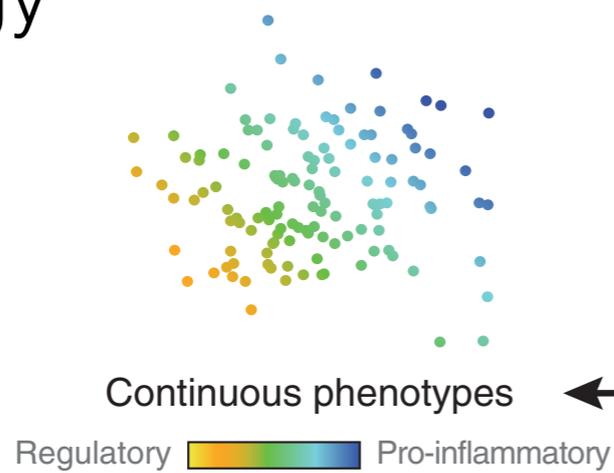
Cell-to-cell variation: topology

Levine et al., Cell (2015), Xu et al. Bioinf (2015) ...
set of dots topology

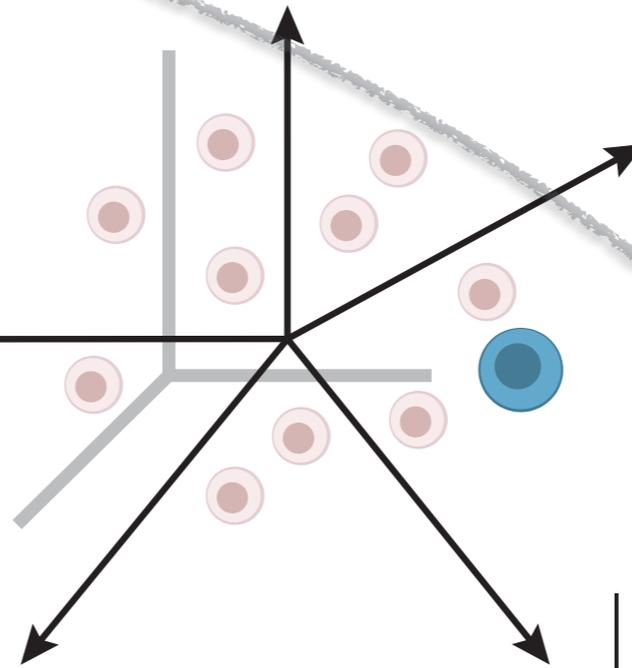


Trapnell et al., Bendall et al. (2014), Haghverdi et al. (2016) ...

line topology

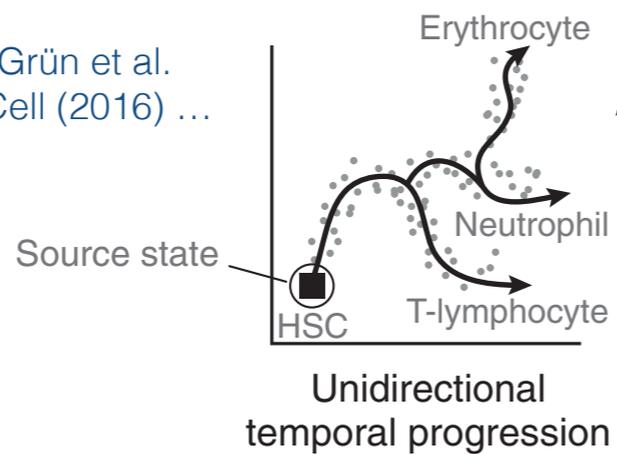


Rizvi et al., Nat Biotechn (2017)
connected topologies

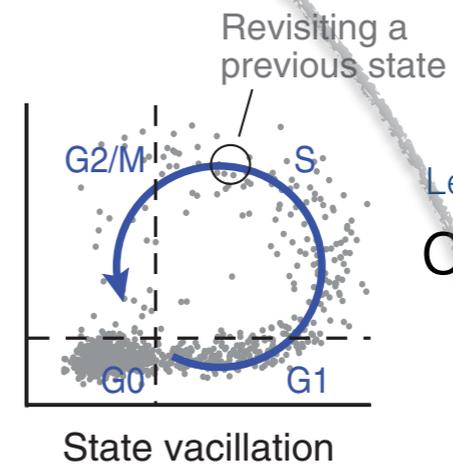


Qiu et al., Nat Meth (2017), Grün et al. Nature (2015) & Cell Stem Cell (2016) ...

tree topology



Leng et al., Nat Meth (2015)
cycle topology



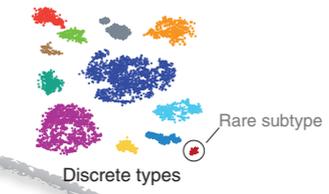
set of dots topology

Need to unify...

... as single-cell data has complicated connected and disconnected topology

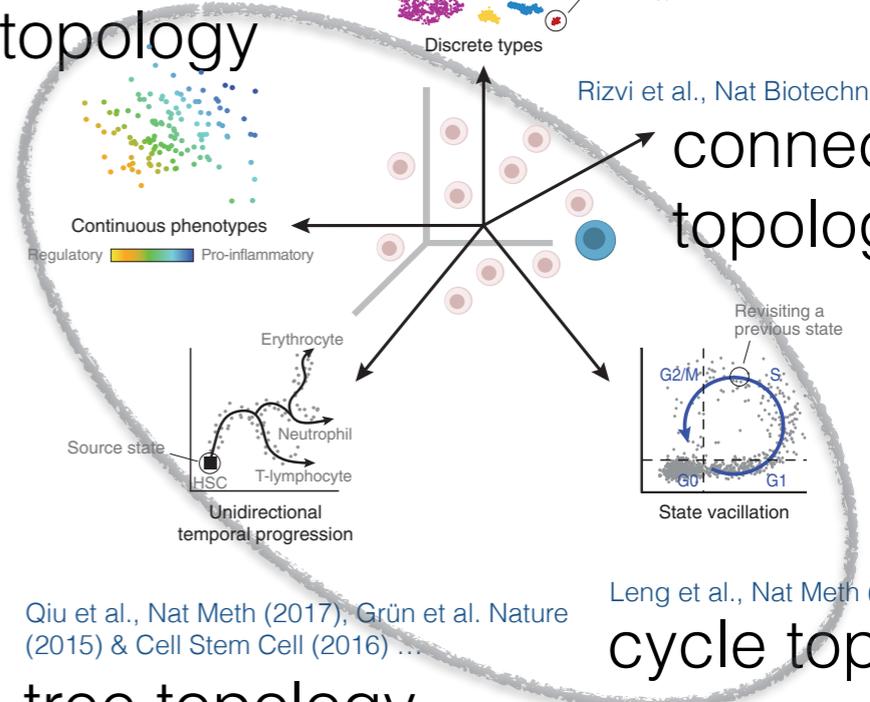
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line topology



Rizvi et al., Nat Biotechn (2017)

connected topologies

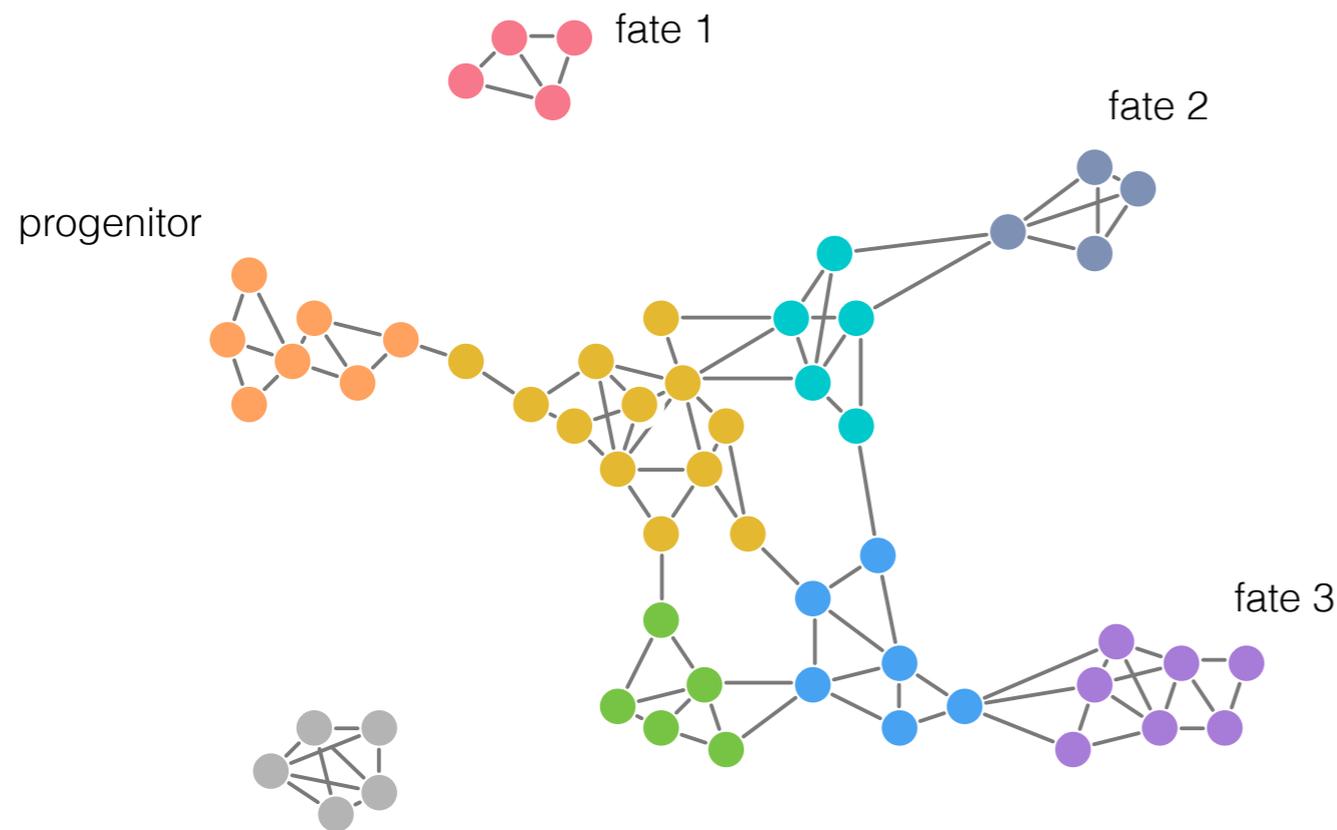


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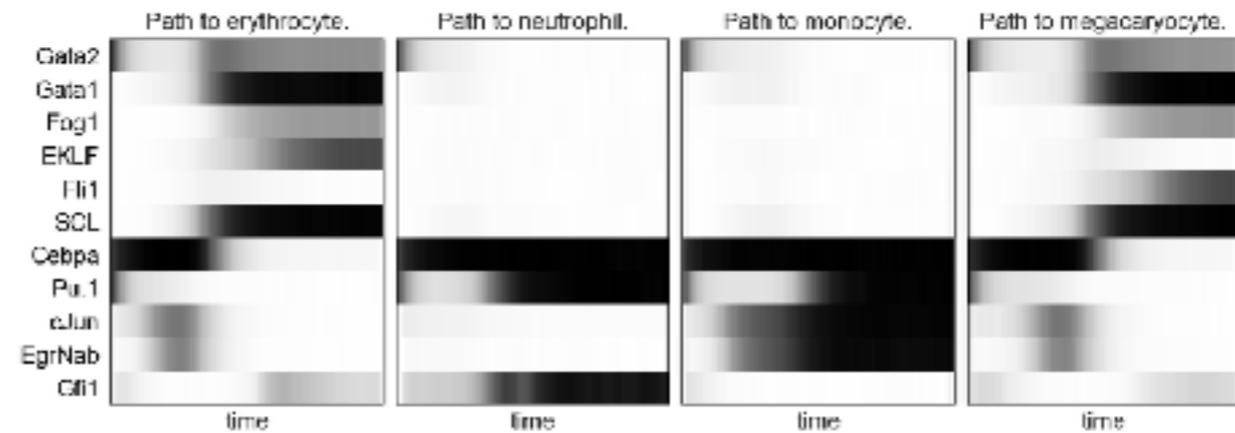
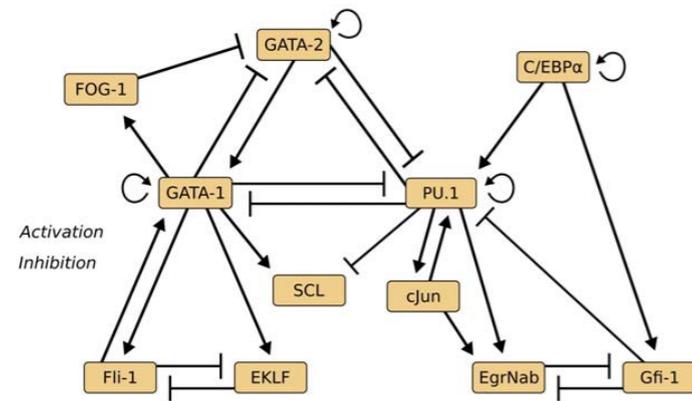
'single-cell graph' represents topology at single-cell resolution

Plan

Simplify single-cell graph to generate a cell map that represents topology at a coarse-grained, human-interpretable resolution.

Graph-based visualization

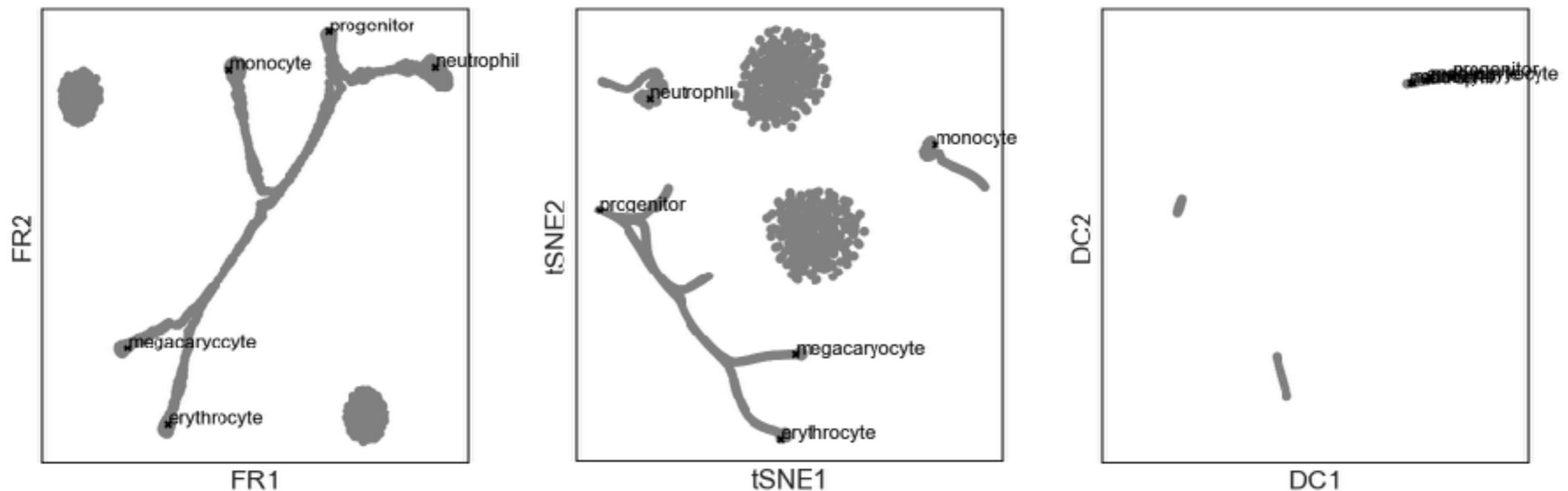
For illustration, model myeloid differentiation...



... and add clusters to the data to model imperfect sampling.

Graph-drawing often conserves topology of single-cell graph.

Weinreb et al., bioRxiv (2017)



Graph-based coordinates

Continuous coordinate: random-walk based distance on graph

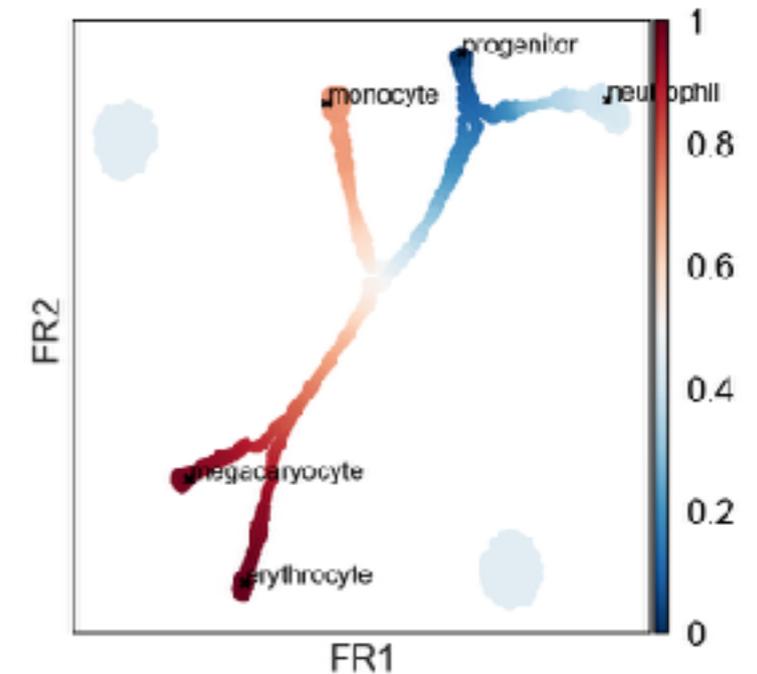
Generalize scale-free random-walk based distance measures to disconnected graphs

$$\text{mean commute time}(\iota_1, \iota_2) = 2n_{\text{edges}} \sum_{r=1+n_{\text{comps}}}^{n_{\text{nodes}}} \left(\frac{1}{1-\lambda_i} \right)^2 (v_{r\iota_1} - v_{r\iota_2})^2,$$

extends Lovász, *Combinatorics* (1993)

$$\text{dpt}(\iota_1, \iota_2) = \sum_{r=1+n_{\text{comps}}}^{n_{\text{nodes}}} \left(\frac{\lambda_i}{1-\lambda_i} \right)^2 (v_{r\iota_1} - v_{r\iota_2})^2,$$

extends Haghverdi *et al.*, *Nat. Meth.* (2016)



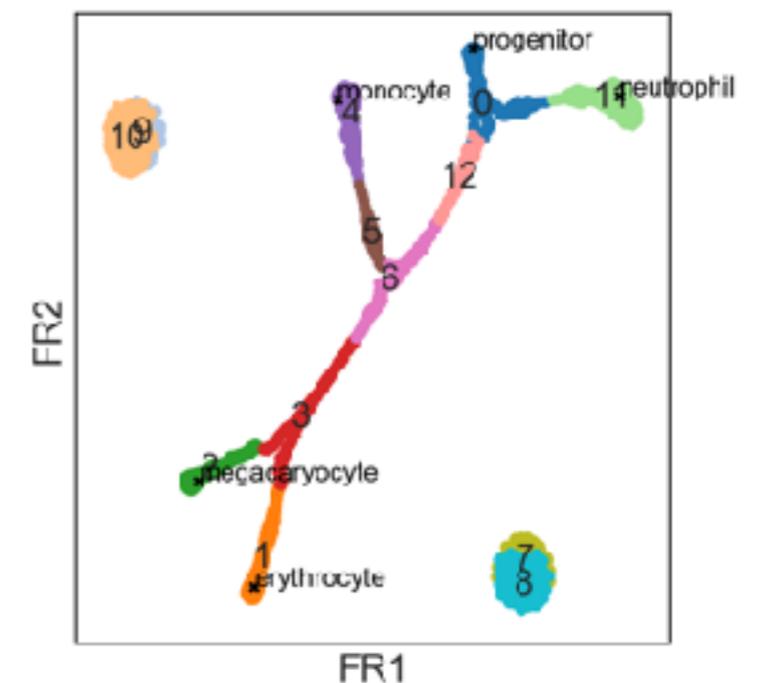
Categorical coordinate: cluster index

Optimizing graph modularity is sensitive to changes in topology

Newman, *Phys. Rev. E* (2004)

Blondel *et al.*, *J. Stat. Mech.* (2008)

Levine *et al.*, *Cell* (2015)



Relating clusters with each other

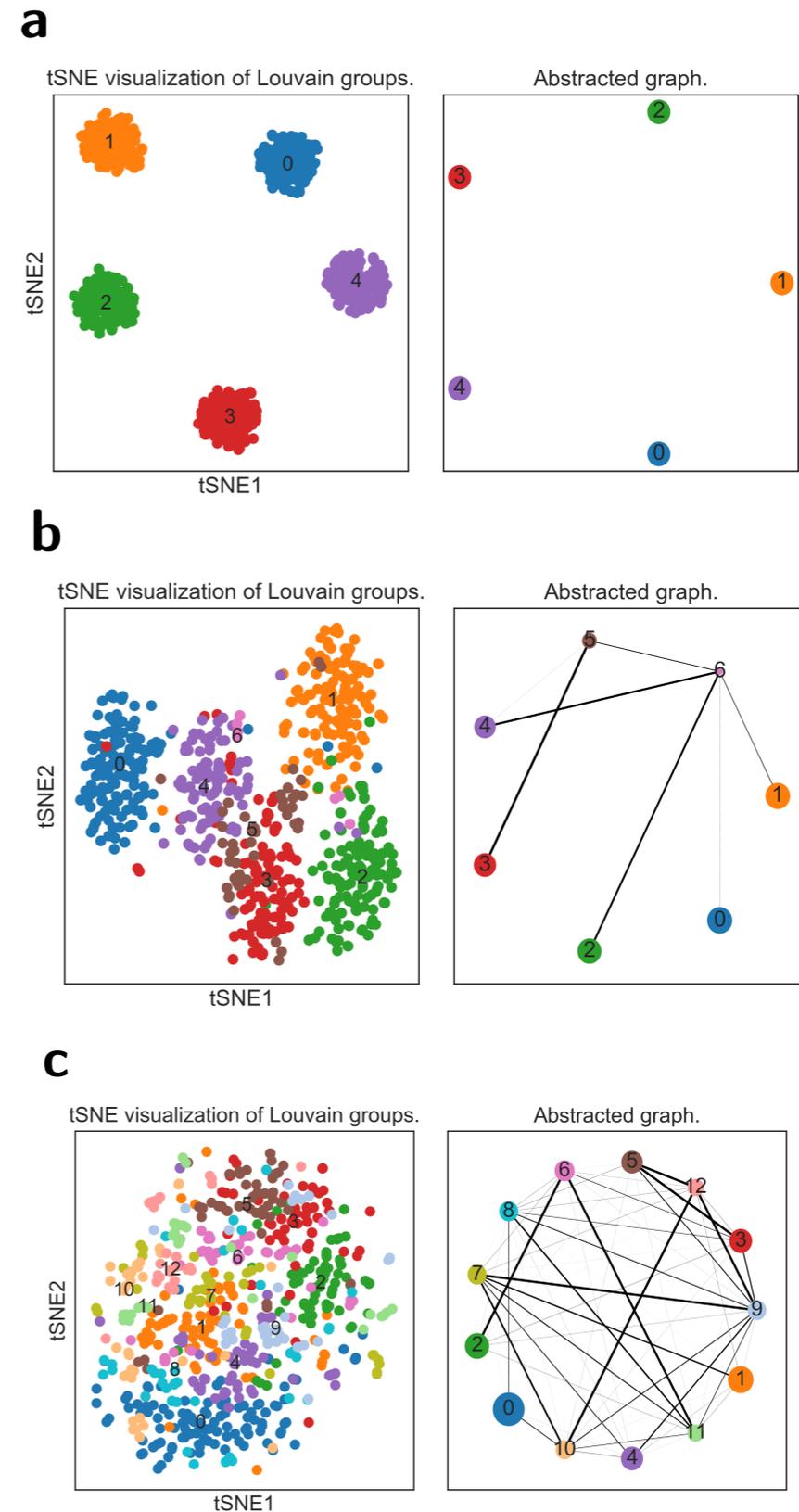
Develop statistical test of connectedness of clusters

Consider clusters as connected if they have more inter-edges than expected under random connections

$$M_{ij} = K_{ij}/n_{\text{edges}} - \theta_i\theta_j$$

$$E[M_{ij}] = 0$$

$$\text{var}[M_{ij}] = \theta_i\theta_j(1 - \theta_i\theta_j)/n_{\text{edges}}$$



Relating clusters with each other

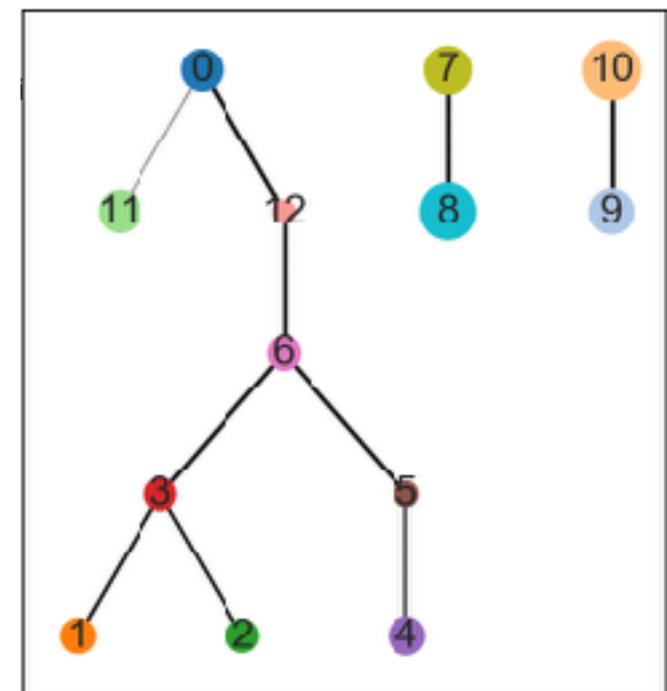
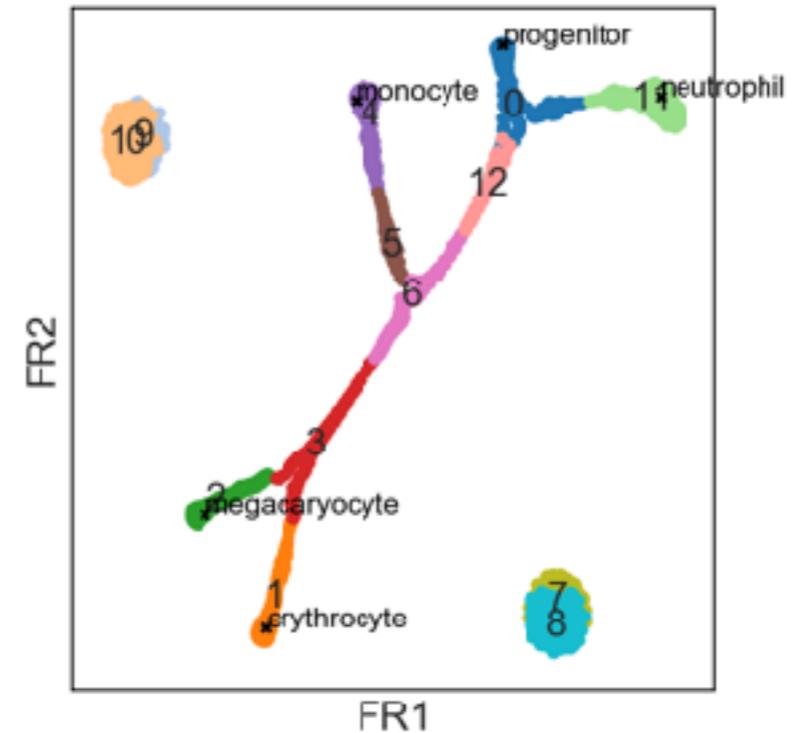
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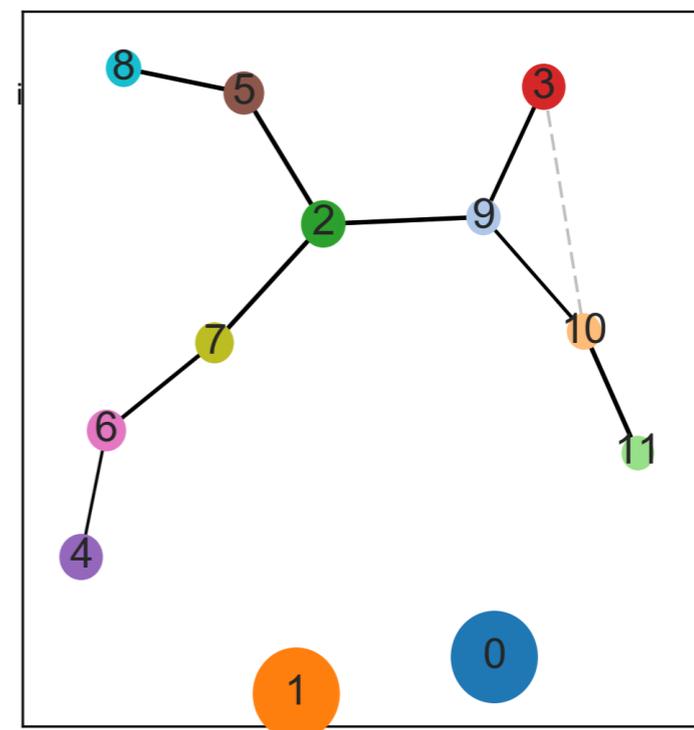
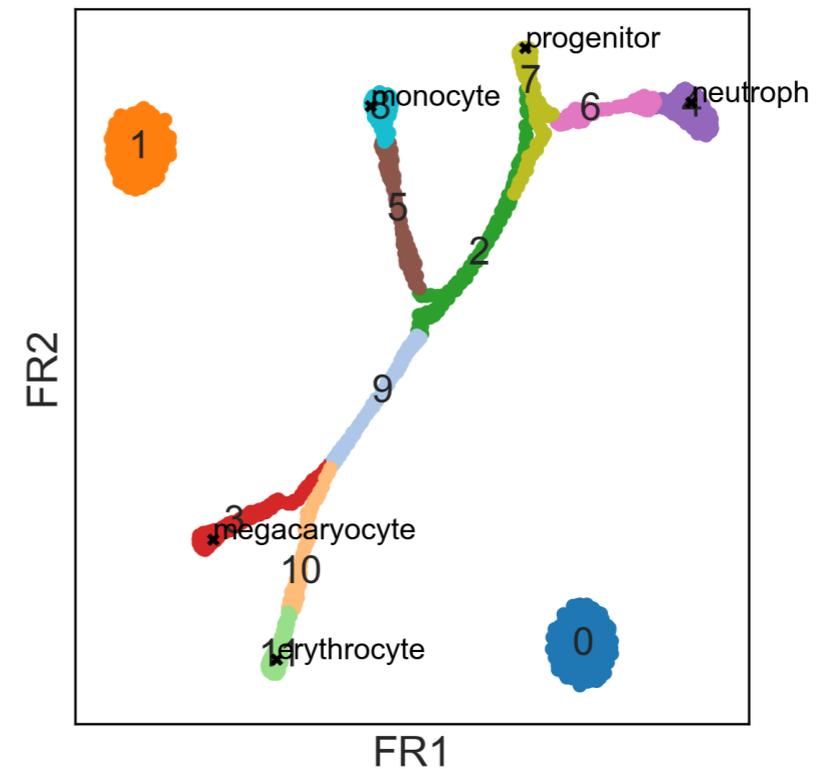
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Relating clusters with each other

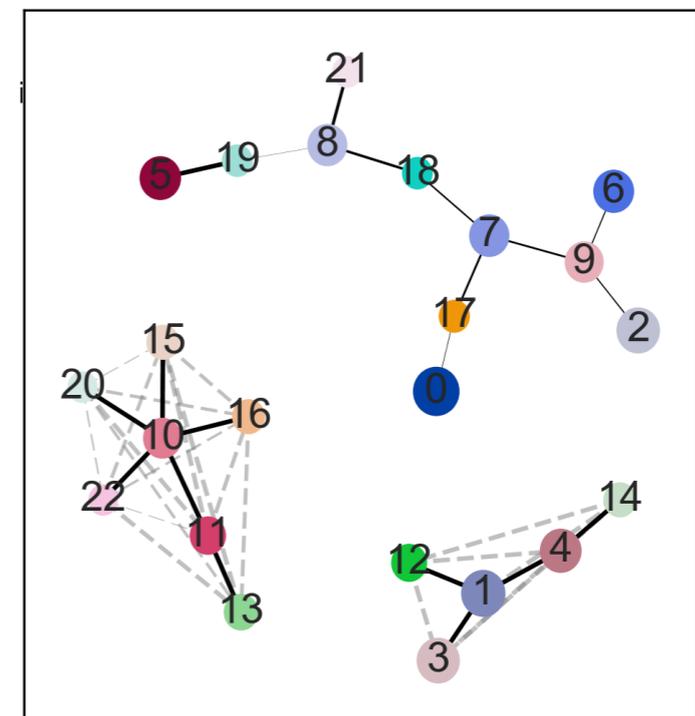
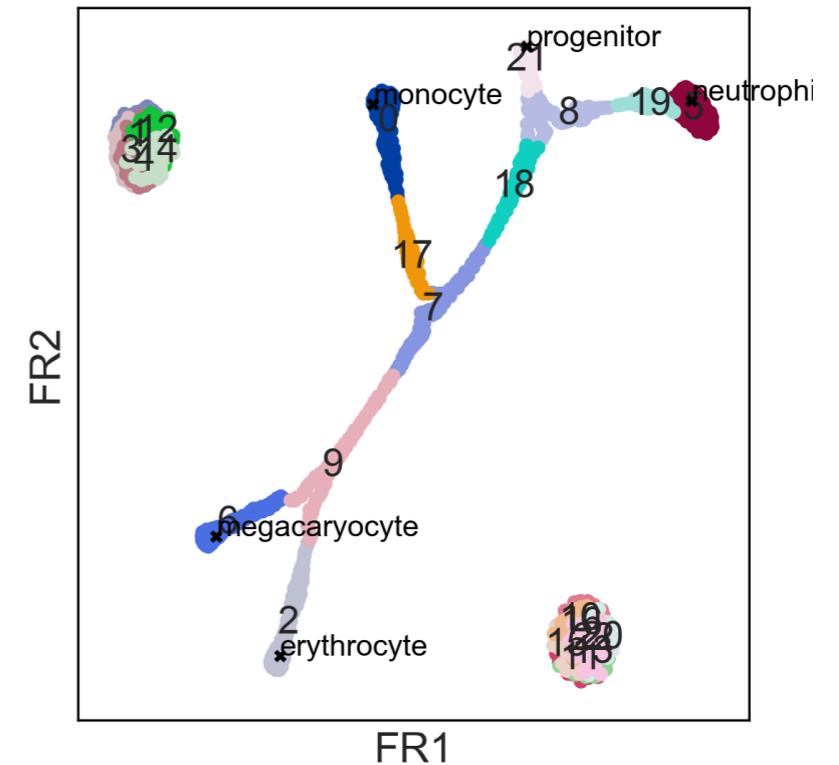
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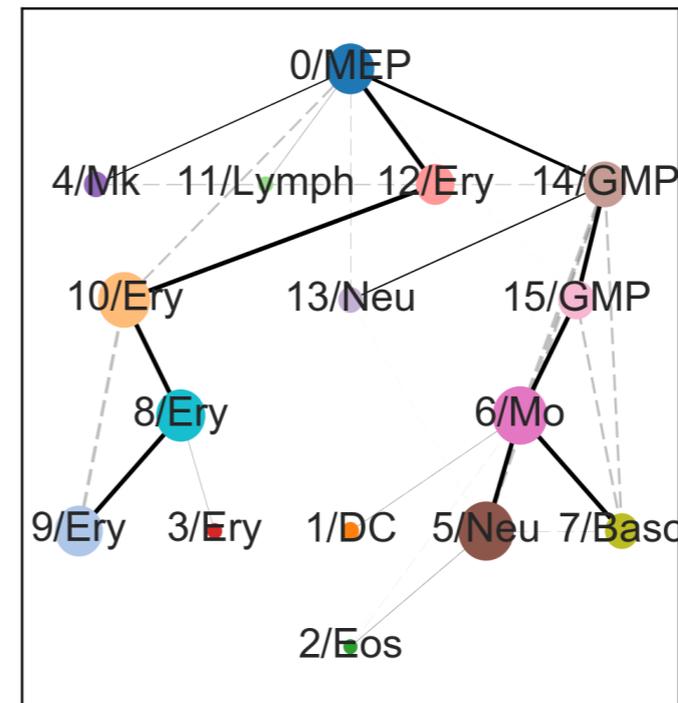
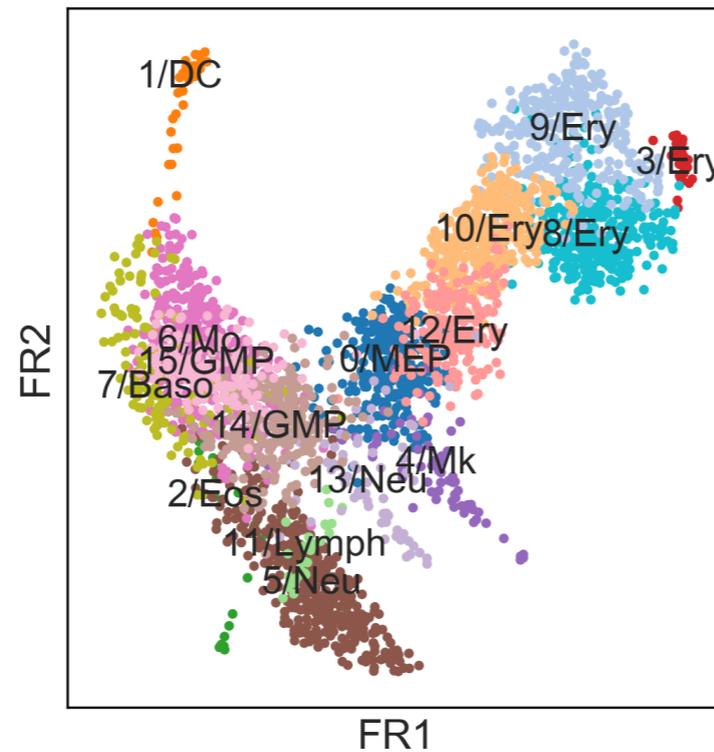
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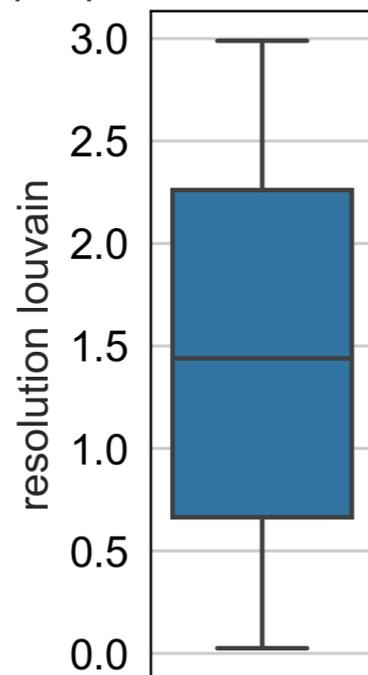
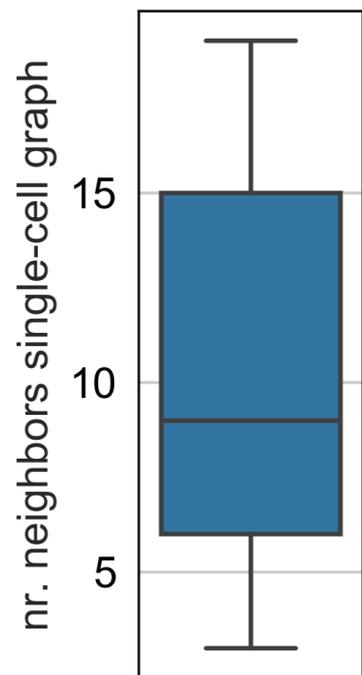


Abstracted topology is robust

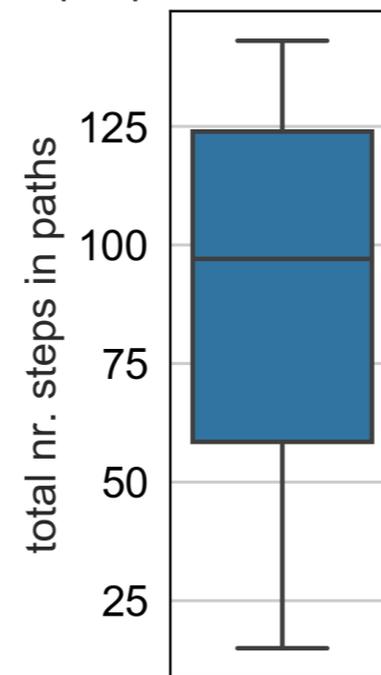
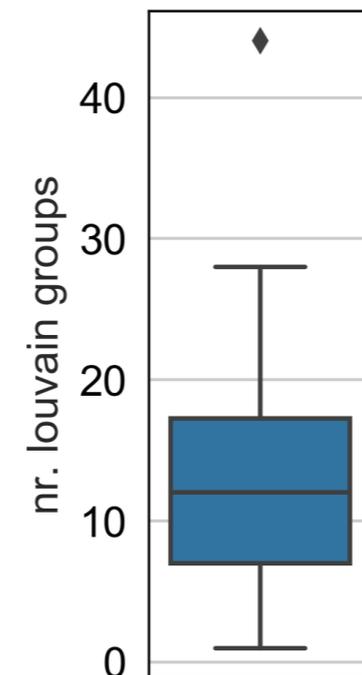


data of
Paul *et al.*, Cell (2015)

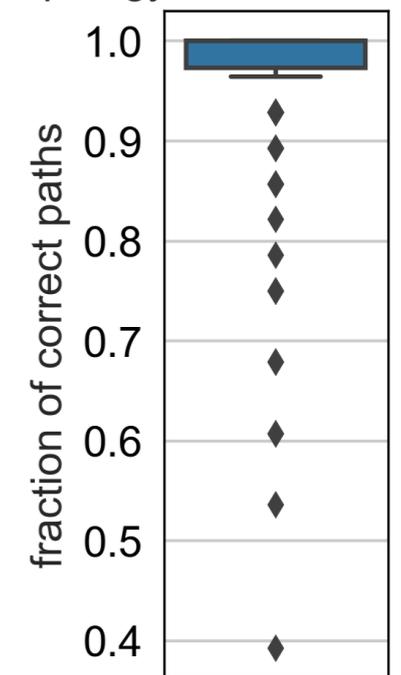
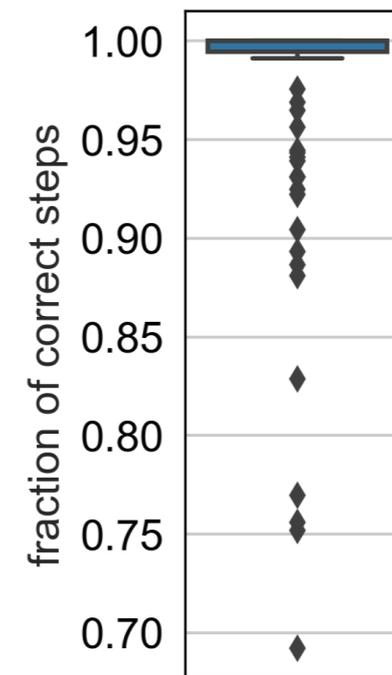
distribution of input parameters



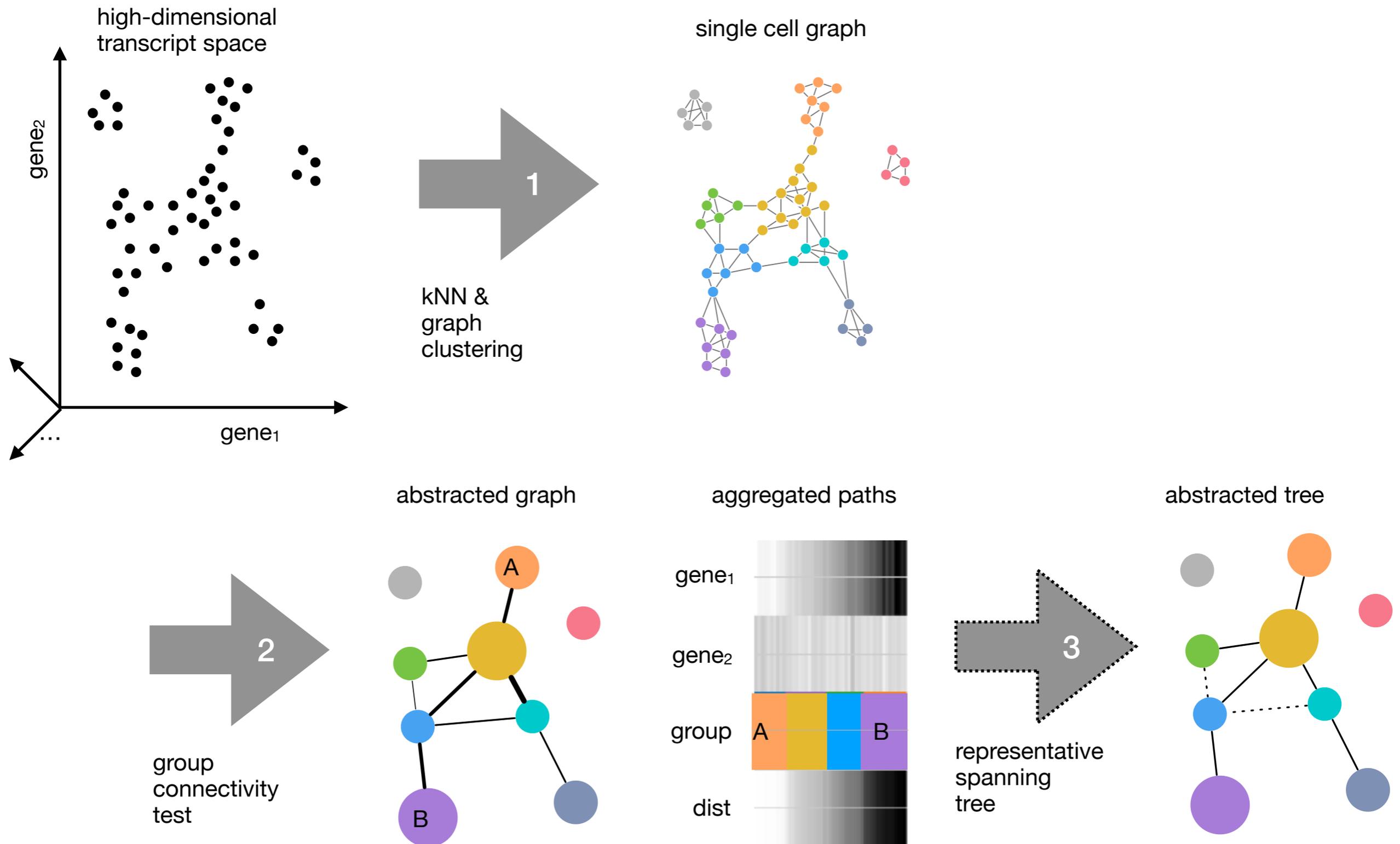
distribution of output parameters



robustness of topology inference



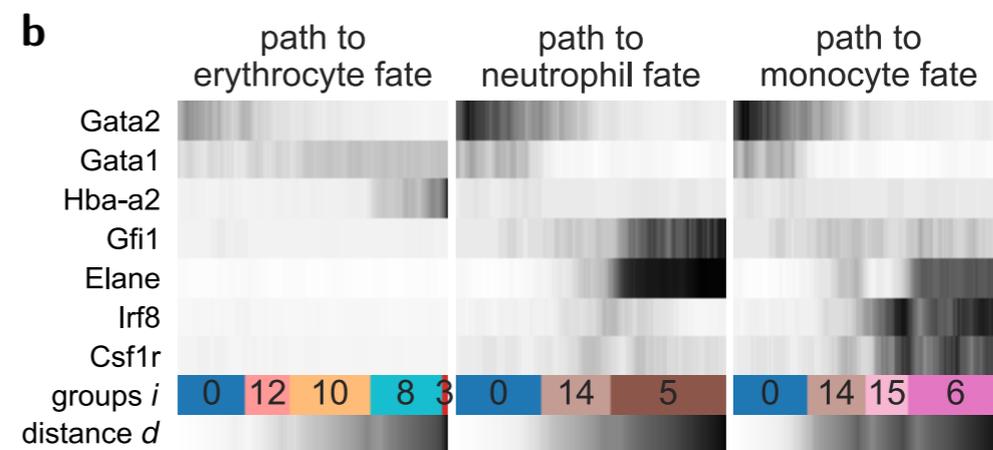
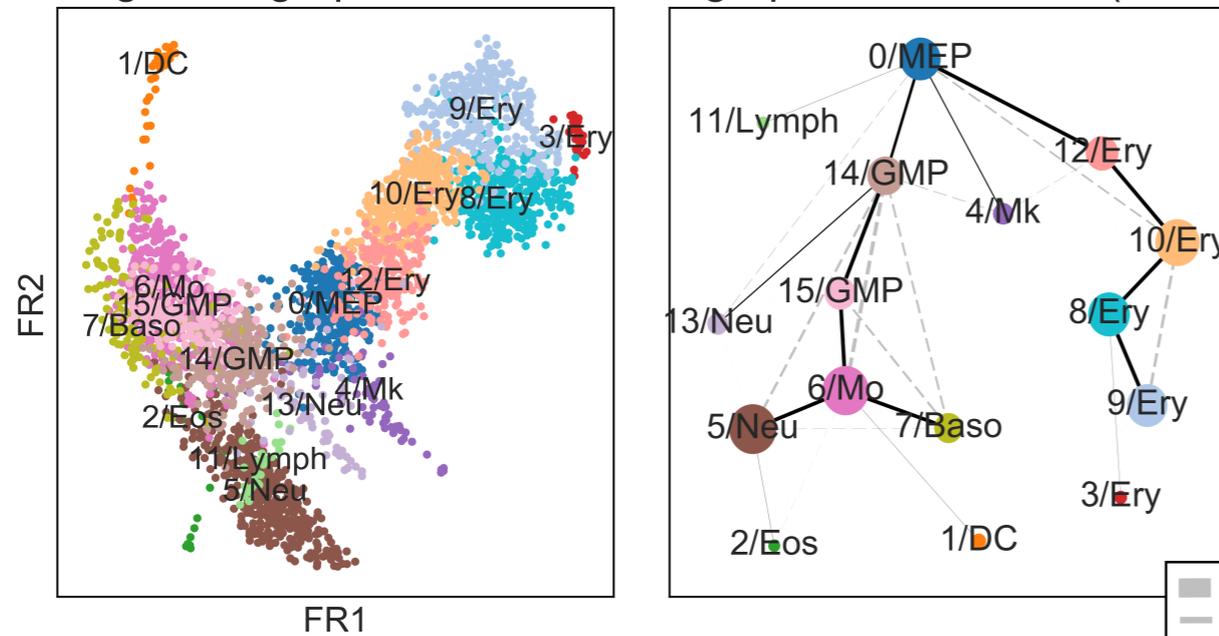
Graph abstraction: overview



Consistent continuous gene changes

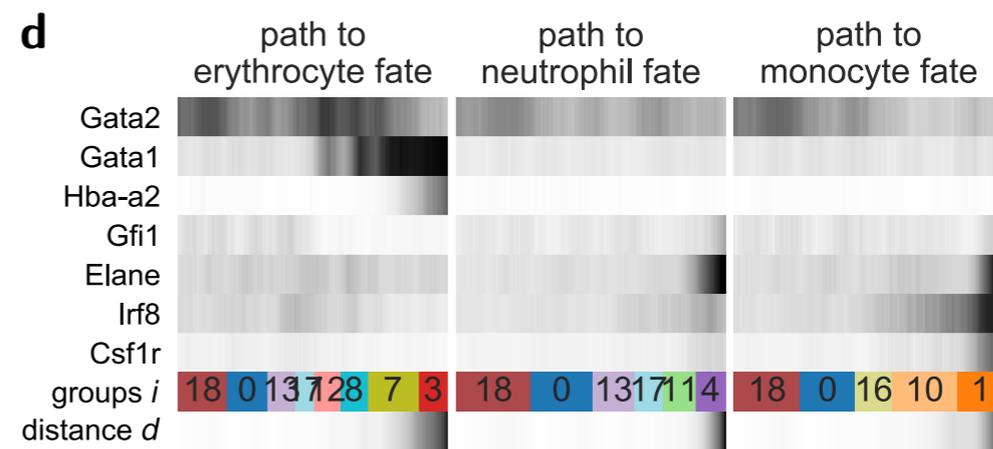
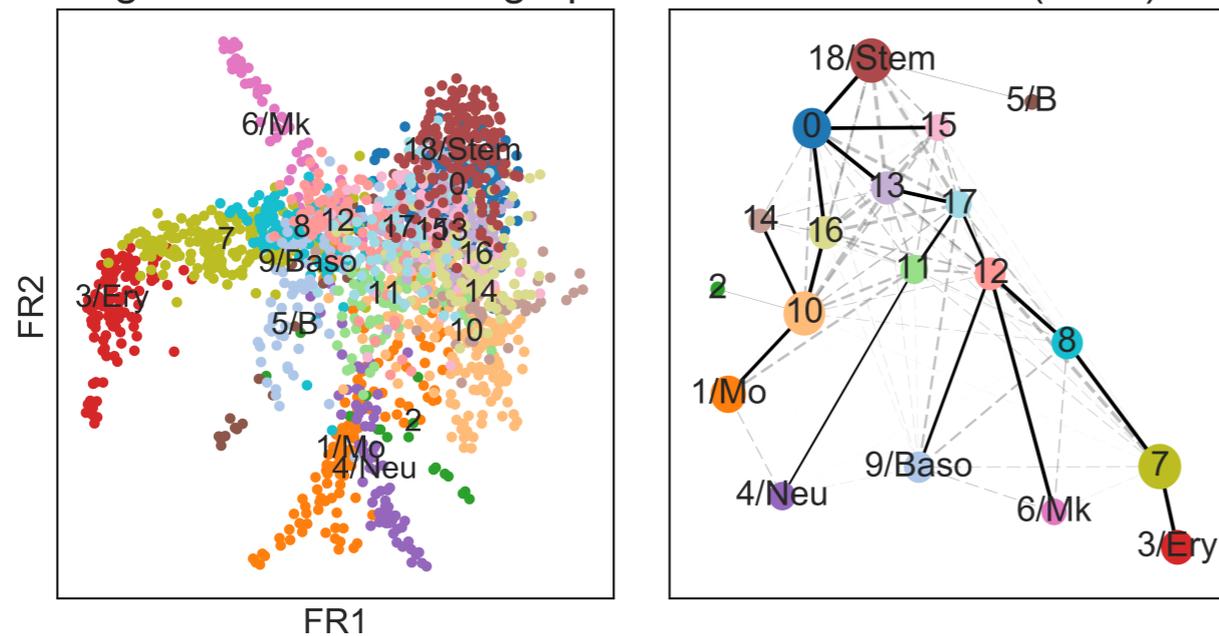
... across datasets from different labs.

a single-cell graph and abstracted graph for Paul et al. (2015)



high confidence
 low confidence
 tree subgraph

c single-cell/ abstracted graph for Nestorowa et al. (2016)



Paul et al., Cell (2015)

Nestorowa et al., Blood (2016)

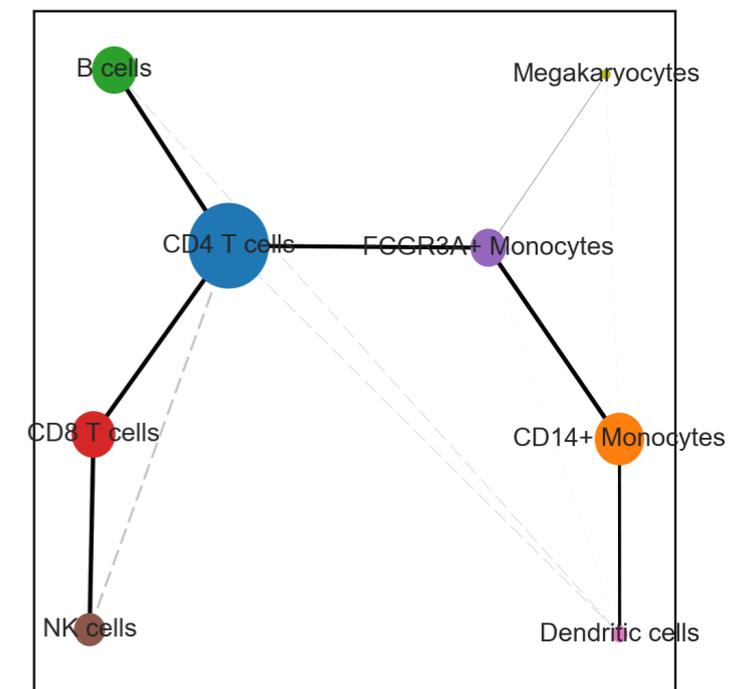
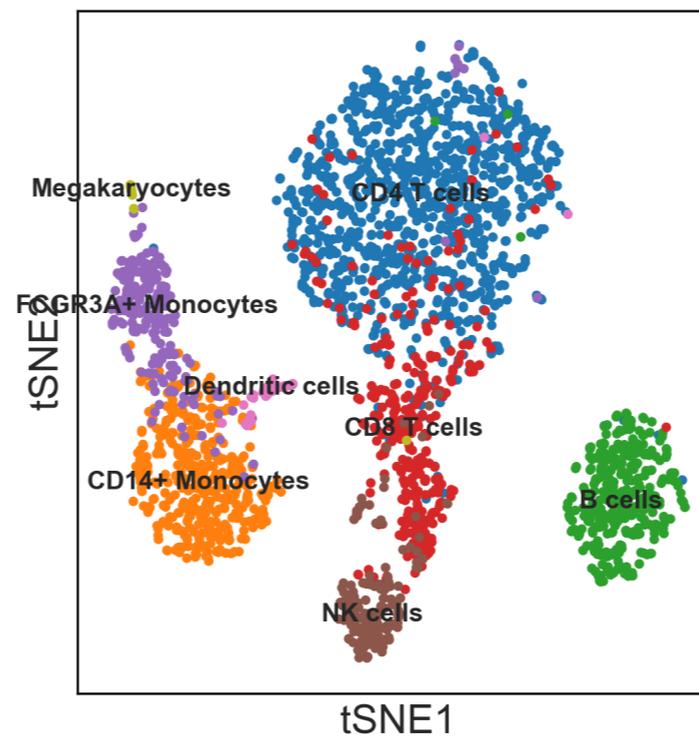
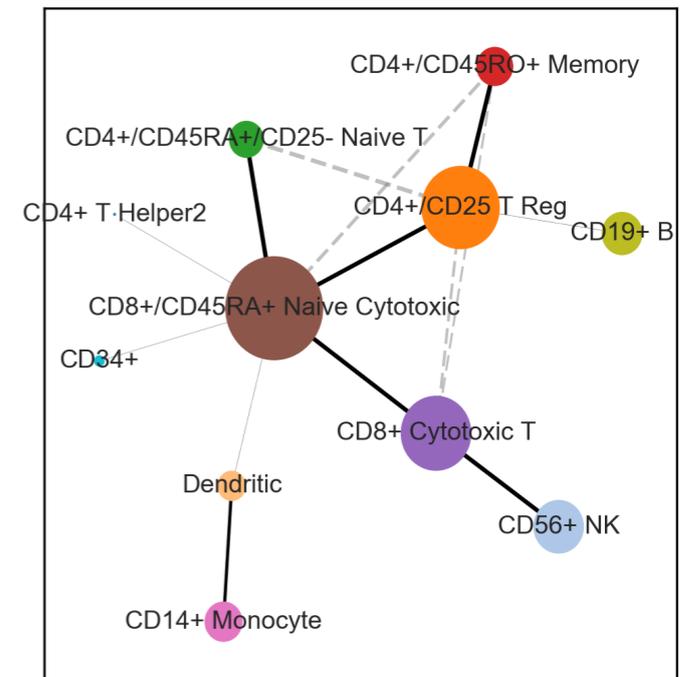
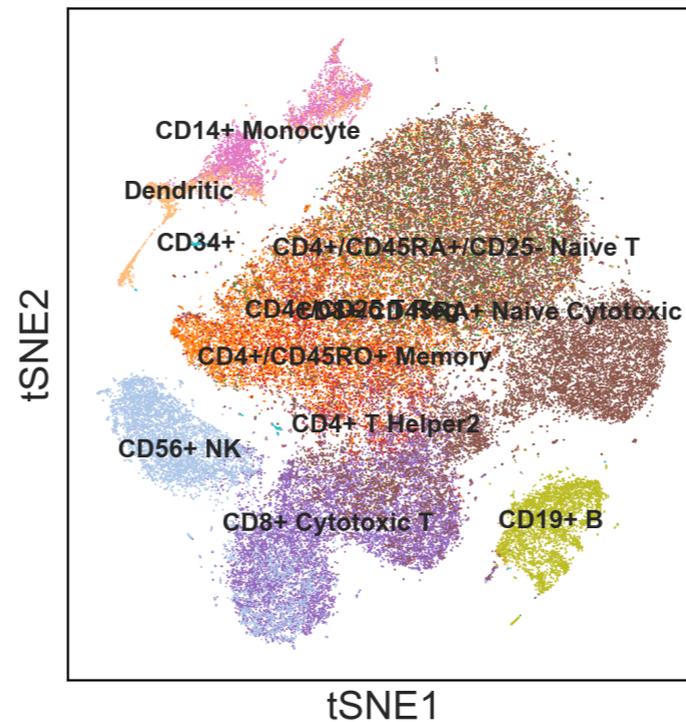
Learn where data is connected

Partial reconstruction of the PBMC lineage tree.

Only motifs can be recovered as data mostly consists of differentiated cells.

data for 68k cells from Zheng *et al.*, Nat. Comms. (2017)

data for 3.6k cells from 10X Genomics



Thanks to

Helmholtz Munich, Theis Lab

Sophie Tritschler

Lukas Simon

Fabian Theis

HelmholtzZentrum münchen

Deutsches Forschungszentrum für Gesundheit und Umwelt

Cambridge U, Göttgens Lab

Fiona Hamey

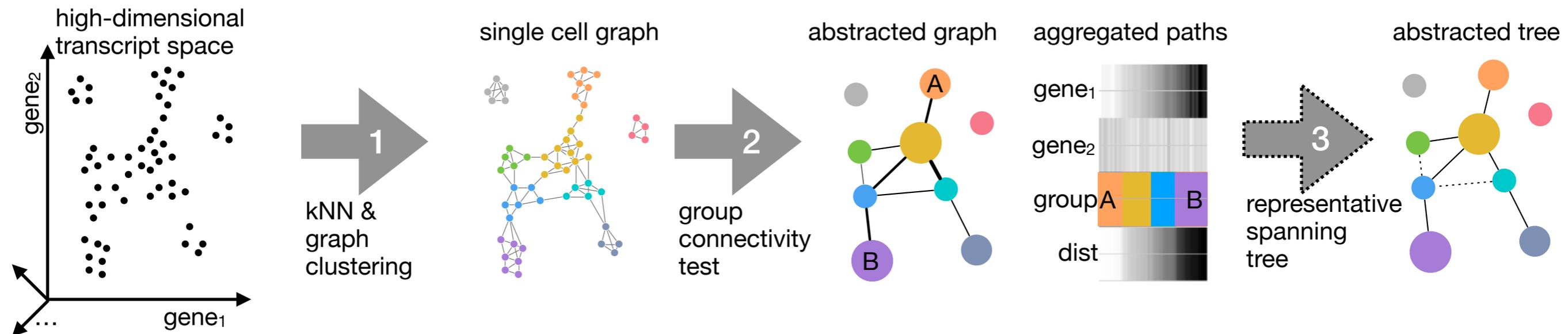
MDC Berlin, Rajewski Lab

Mireya Plass

Jordi Solana

Nikolaus Rajewski

Thank you for your attention!

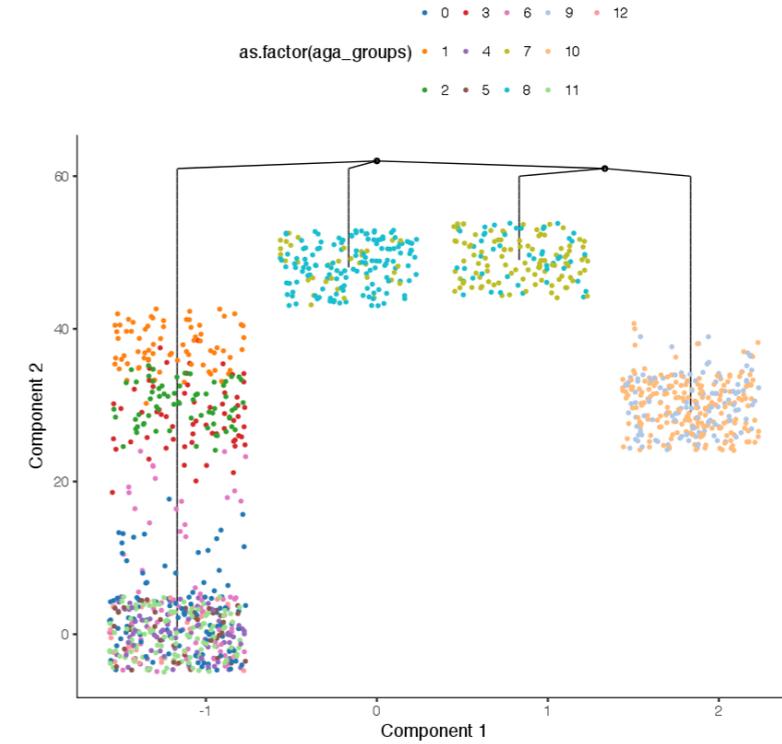
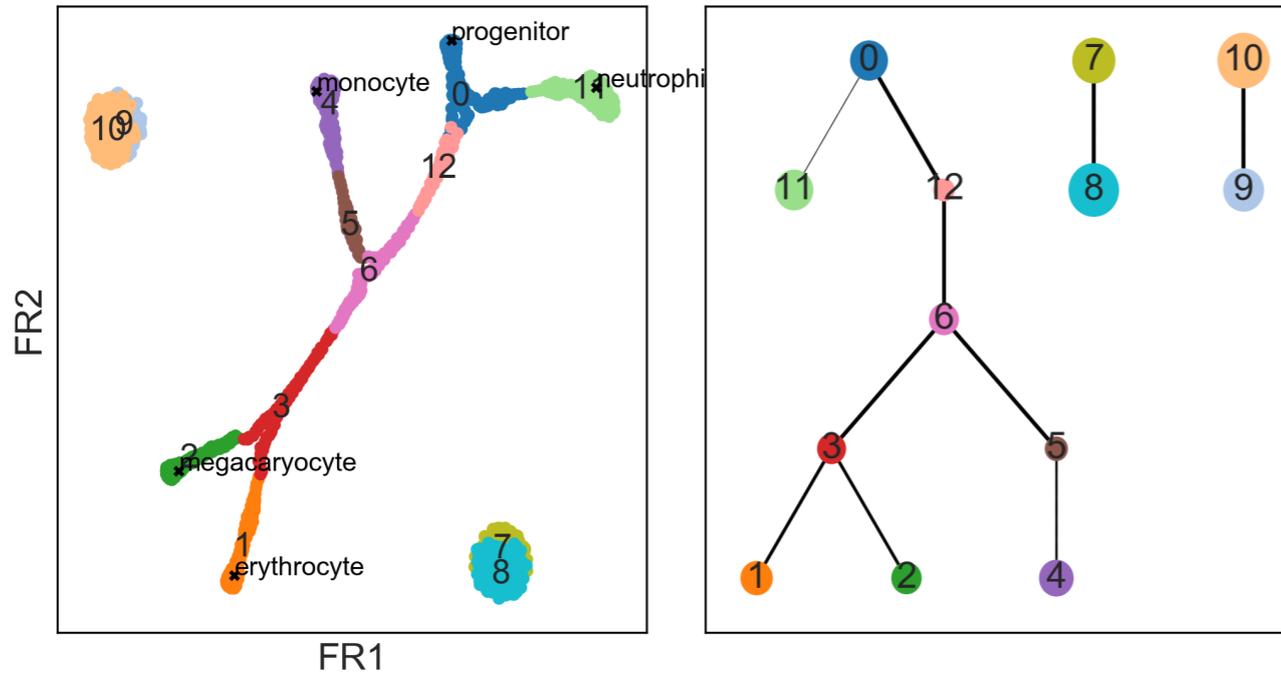


Code and documentation: https://github.com/theislab/graph_abstraction

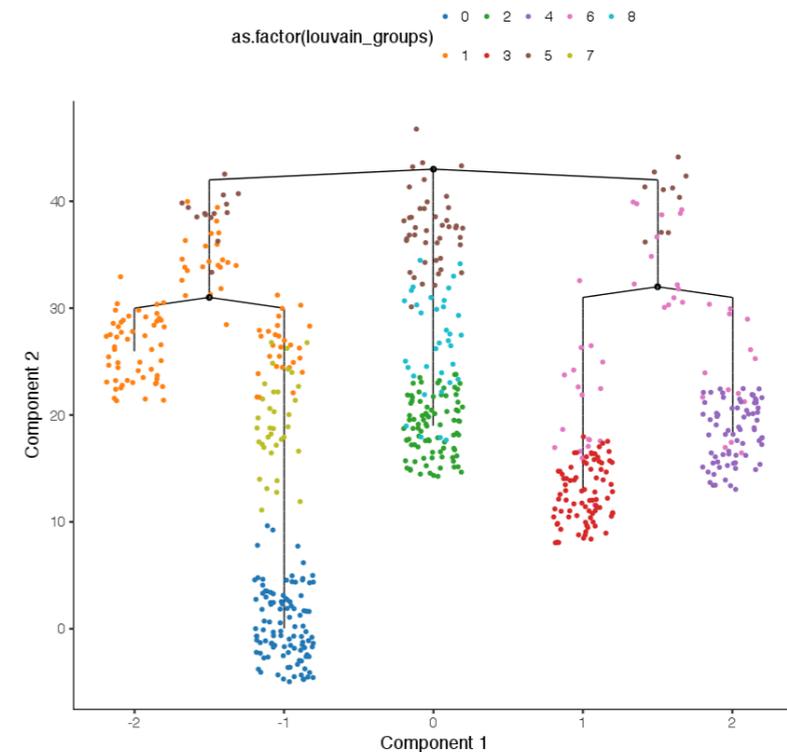
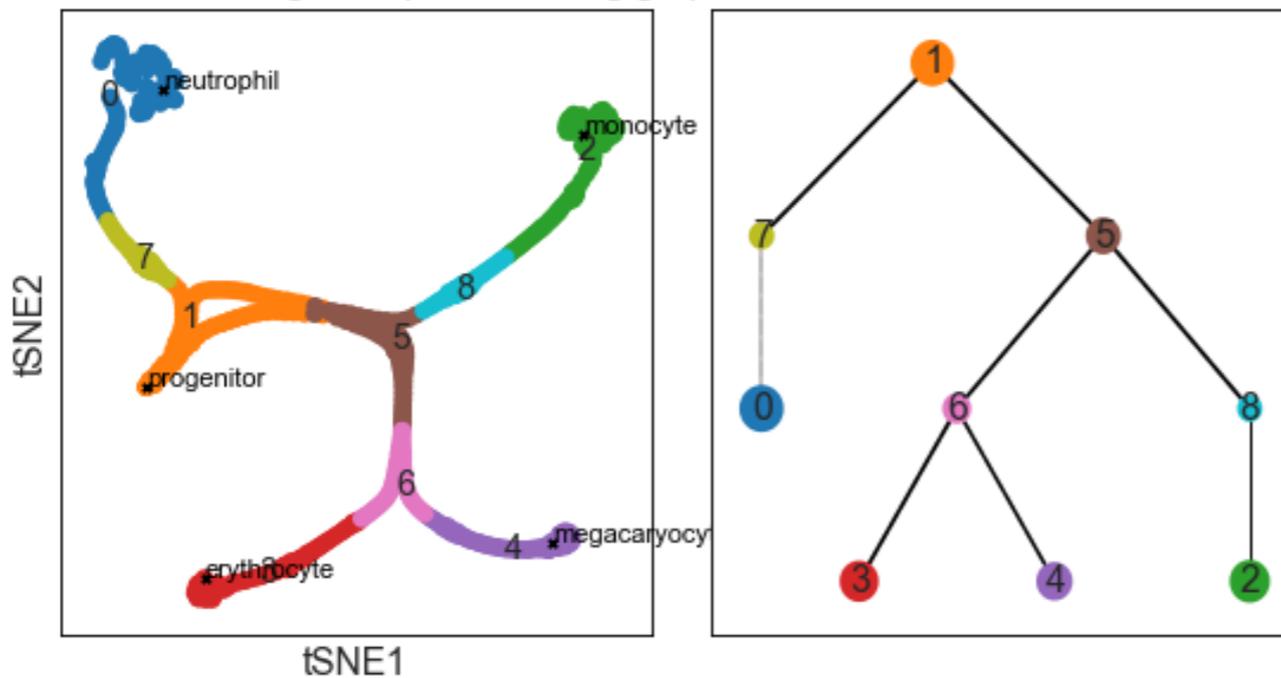
On bioRxiv within the next days

Comparison with Monocle

Single-cell graph \mathcal{G} of minimal example. Abstracted graph \mathcal{G}^* relating groups.



Reconstructing a simple tree using graph abstraction.



Comparison with stemID

Grün et al., Nature (2016)
Grün et al., Cell Stem Cell (2017)

Single-cell graph \mathcal{G} of minimal example. Abstracted graph \mathcal{G}^* relating groups.

